Exercise 1.11

Neptunium. In the fall of 2002, a group of scientists at Los Alamos National Laboratory determined that the critical mass of neptunium-237 is about 60 kg. The critical mass of a fissionable material is the minimum amount that must be brought together to start a chain reaction. This element has a density of 19.5 g/cm^3. What would be the radius of a sphere of this material that has a critical mass?

Solution

The relationship between mass and volume is given by

$$m = \rho V,$$

where ρ is the density. Substitute $V = (4/3)\pi r^3$ for the volume of a sphere.

$$m = \rho\left(\frac{4}{3}\pi r^3\right)$$

Solve for r.

$$3m = 4\pi\rho r^3$$

$$r^3 = \frac{3m}{4\pi\rho}$$

$$r = \sqrt[3]{\frac{3m}{4\pi\rho}}$$

Now that the formula for r is known, substitute the given quantities and convert them to SI units.

$$r = \sqrt[3]{\frac{3(60\ \cancel{\text{kg}})}{4\pi\left[19.5\ \frac{\cancel{\text{g}}}{\cancel{\text{cm}^3}} \times \frac{1\ \cancel{\text{kg}}}{1000\ \cancel{\text{g}}} \times \left(\frac{100\ \cancel{\text{cm}}}{1\ \text{m}}\right)^3\right]}} \approx 0.09\ \text{m}$$

Note that it's unclear whether the tens or ones place in the given mass, "60 kg," is uncertain. Here it's assumed the tens place is uncertain so that there's only one significant figure. Scientific notation is needed to be specific.